Introducing a Practical Educational Tool for Correlating Algorithm Time Complexity with Real Program Execution

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Abstract. Algorithm time complexity is an important topic to be learned for programmer; it could define whether an algorithm is practical to be used on real environment or not. However, learning such material is not a trivial task. Based on our informal observation regarding students’ test, most of them could not correlate Big-O equation to real program execution. This paper proposes J-CEL, an educational tool that acts as a supportive tool for learning algorithm time complexity. Using this tool, user could learn how to correlate Big-O equation with real program execution through intuitive graphics (resulted from providing three components: Java source code, source code input set, and time complexity equations). According to our evaluation, students feel that J-CEL is helpful for learning the correlation between Big-O equation and real program execution. Further, the use of Pearson correlation in J-CEL shows a promising result.

1 Introduction

Algorithm time complexity indicates how fast an algorithm runs in regard to input size [1]. It is an important topic to be learned for programmer; it could define whether an algorithm is practical to be used on real environment. Therefore, it is listed as a required topic to be mastered for undergraduates in Computer Science (CS) curricula [2].

At CS undergraduate major, algorithm time complexity is usually taught theoretically using Big-O notation; where operations on given algorithm are correlated to input size through abstracted pseudo-execution. It is true that such notation is helpful for learning time complexity. However, according to our informal observation, some students find it difficult to correlate Big-O equation to real program execution. They need some real examples regarding to that correlation.

In response to aforementioned gap, this paper proposes an educational tool for learning algorithm time complexity by examples. Different with a tool mentioned in [3], our tool could be used without deep technical knowledge about programming language. In other words, it is more practical to be used in real environment. In addition to proposing a tool, we also observe the characteristics of Pearson correlation [4] (i.e., an algorithm used in proposed tool to correlate resulted equation with real program execution).
2 Related Works

In general, despite various terminologies used, Computer Science (CS) educational tool can be classified into twofold: programming-oriented and algorithm-oriented educational tool [3, 5].

Programming-oriented educational tool puts more stress on technical knowledge such as how to write a program. At CS undergraduate major, it has been frequently used for learning Introductory Programming; such tool is proved to be effective for teaching undergraduate students [6–9]. Most programming-oriented educational tools are either program visualization or visual programming tool. On the one hand, program visualization tool is focused on simulating how a program works. It is usually featured with a rich information regarding running program (e.g., variable state). Several examples of program visualization tool are PythonTutor [10], OmniCode [11], SRec [12], Jelliot 3 [13], JIVE [14], and VILLE [15]. On the other hand, visual programming tool is focused on preventing errors caused by writing the code directly. It is usually featured with indirect interaction mechanism between user and target program. Several examples of visual programming tool are Raptor [16], SFC Editor [17], Scratch [18], Alice [19] and Greenfoot [20].

Algorithm-oriented educational tool puts more stress on algorithmic steps. At CS undergraduate major, it has been frequently used to cover complex topic such as data structures [21, 22] and algorithm strategies [3, 23–26]. When perceived from how they focus on visualization, algorithm-oriented educational tools can be further classified into twofold: algorithm visualization and standard algorithm-oriented educational tool. On the one hand, algorithm visualization tool focuses on utilizing visualization as its main interaction feature. Different with program visualization tool, it involves no specific programming language. In other words, it displays more abstract information than program visualization tool. Several examples of such tool are VisuAlgo [22], AP-ASD1 [21], AP-SA [23], and AP-BB [24]. On the other hand, standard algorithm-oriented educational tool does not focus on visualization as its main feature. Several examples of such tool are GreedEx [26], GreedExCol [25], and Complexitor [3].

Algorithm time complexity is a metric used to measure the time growth of a particular algorithm toward input size [1]. In other words, it could be used to define whether a particular algorithm is practical to be used on real environment. Despite its benefit, learning algorithm time complexity is not a trivial task. According to our informal observation regarding students’ test toward that topic, most of them could not grasp the idea behind calculating algorithm time complexity using Big-O notation. They could not correlate Big-O equation to real program execution. Further, an informal survey provided by [3] also shows similar phenomenon.

Complexitor [3] is an educational tool specifically designed to learn the correlation between Big-O equation and real program execution. User can learn how algorithm time complexity is defined based on the number of processes generated from empirical execution. Considering source code used to generate the number of processes can be written in any programming languages, Complexitor separates programming-language-dependent syntaxes (to count the number of processes) from its system wherein such syntaxes should be embedded prior execution. It is true that this mechanism enables Complexitor to work with any programming languages. However, it indirectly enforces user to know a lot of technical details regarding target programming language. Such requirement may be troublesome for users with limited programming capability.
3 Proposed Methodology

This paper proposes an educational tool that acts as a supportive tool for learning algorithm time complexity. It is named J-CEL (Java-based time Complexity Educational tool). Using this tool, user could learn how to correlate Big-O equation with real program execution by providing three components: arbitrary source code (written in Java programming language), source code input set, and time complexity equations.

It is true that Complexitor [3], at some extent, also aims similar goal as ours. However, we would argue that our proposed tool is more practical to be used in real environment: user is not required to have deep technical knowledge regarding target programming language since programming-language-dependent syntaxes are automatically generated. Further, we also provide richer information regarding time complexities and their respective correlation.

The correlations between time complexity equation and real programming execution are calculated using methodology given on Fig. 1. It contains four phases which are embedding, execution, generation, and correlation phase.

An Arbitrary Source Code  \(\rightarrow\)  Embedding Phase

\[\rightarrow\text{Embedded Source Code}\]

Source Code Input Set  \(\rightarrow\)  Execution Phase

\[\rightarrow\text{Empirical Sequence}\]

Time Complexity Equations  \(\rightarrow\)  Generation Phase

\[\rightarrow\text{Theoretical Sequences}\]

Correlation Phase  \(\rightarrow\)  Correlation Result

![Diagram of the proposed methodology](image)

Embedding phase will embed a counter mechanism to calculate how many processes required for executing given code toward an input stream. It accepts an arbitrary source code as its input and generates embedded source code as its output. Basically, this phase works in similar manner as in [3] by incorporating three kinds of statement: counter initialization, counter increment, and counter print statement. First, counter initialization will be embedded at the beginning of main method. Second, counter increment will be embedded at the end of each statement. Third, counter print statement will be embedded at the last statement of main method. Further detail and
example about that counter mechanism can be seen in [3].

To automatically embed statements, ANTLR [27] is used to parse inputted source code and locate desired positions. Since ANTLR requires different lexer and parser for each programming language, we limit our target programming language to Java as our case study.

Execution phase will generate an empirical sequence (i.e., a sequence that represents the number of process for real program execution) by executing embedded source code multiple times with respect to source code input set. Empirical sequence will be used for correlating time complexity with real program execution at correlation phase. Execution phase works in twofold. At first, embedded source code will be compiled to a program. Later, each input stream from source code input set will be fed to given program; each input stream will result an integer representing the number of process toward given stream. These integers will be sorted based on input size in ascending order and returned as an empirical sequence.

Generation phase will generate theoretical sequences with respect to the number of time complexity equations. Each equation will result one theoretical sequence where each integer on given sequence represents the number of process for given complexity toward given input. Theoretical sequences will be used for correlating time complexity with real program execution at correlation phase.

Correlation phase will generate correlation value for each theoretical sequence toward empirical sequence. It will be calculated using Pearson correlation algorithm [4], a well-known algorithm to correlate two sequences. Since the correlation between two constant sequences cannot be detected through Pearson correlation, the correlation between constant theoretical sequence (i.e., \(O(1)\)) and empirical sequence will be calculated by measuring how far the difference between each empirical value and the mean of empirical values. If all differences are lower than 10% of the mean, that empirical sequence will be assumed as constant sequence with 1 as its correlation value. This mechanism is adopted from [3].

To enhance user understanding further, J-Cel is featured with two intuitive User Interfaces (UIs). The first UI will be used to accept three input components while the second one will be used to display resulted correlations.

J-Cel’s default UI for accepting input components can be seen in Fig. 2 while its UI when all input components have been filled can be seen in Fig. 3. It contains five panels which are source code file chooser, input set panel, complexity equation panel, exception line field, and source code display. These panels are referred as A, B, C, D, and E respectively on Fig. 2. First, source code file chooser is used to select an arbitrary source code. Second, input set panel is used to manipulate input set; each input stream should be stored as a text file with input size as its name. If there are two input sizes for one file, it will be separated using a hyphen on filename. Third, complexity equation panel is used to manipulate time complexity equation. For convenience, default time complexity equations (i.e., \(O(1)\), \(O(\log n)\), \(O(n)\), \(O(n \log n)\), \(O(n^2)\), \(O(2^n)\), \(O(n^n)\) and \(O(n!)\)) will be provided. New time complexity equation will be inputted through scripting mechanism; it will be written by following some rules and converted to Java-based statements for generation phase. Involved rules cover how to write power, logarithmic, and factorial equation. Fourth, exception line field is used to mark lines that will be ignored while generating empirical sequence. Such feature can be used to accentuate the correlation between given source code and target algorithm time complexity. Exception lines will be written in comma separated values and they will be excluded automatically from embedding and execution phase with the help of ANTLR [27]. Last, source code display is used to display inputted source
code.

J-CEL’s UI for displaying correlation result can be seen on Fig. 4. It contains four panels which are graph setting panel, correlation graph display, input stream selection panel, and source code display. These panels are referred as A, B, C, and D respectively on Fig. 4.

First, graph setting panel is used to manipulate correlation graph display. User can select which time complexity equation he wants to display. Further, he can also set how many logarithmic layers that will be applied to given graph. Logarithmic layer is used to mitigate the gap between theoretical and empirical sequence when such difference cannot be seen by default: applying logarithmic layer will reduce the difference between two numbers while keeping their comparison order similar (see Fig. 5 for an example about applying a logarithmic layer to a graph displayed on Fig. 4). To provide clearer analysis for users, J-CEL is also featured with a default prediction of time complexity on graph setting panel. It will show time complexity equation which correlation toward empirical sequence is the highest.

Fig. 2. J-CEL’s UI for accepting three input components: an arbitrary source code, source code input set, and time complexity equations.
Fig. 3. J-CEL’s UI for accepting input components when all components are ready.

Fig. 4. J-CEL’s UI for displaying correlation result.
Fig. 5. Correlation graph display from Fig. 4 that has been applied with a logarithmic layer.

Second, correlation graph display is used to show how close the correlation between selected theoretical sequence and the empirical one. Each sequence will be represented with a line; orange line refers to theoretical sequence while the red one refers to empirical sequence. On given display, horizontal axis represents input size while vertical axis represents the number of process; similar line equation refers to high correlation between theoretical and empirical sequence (see Fig. 4 for an example of high correlation and Fig. 6 for an example of low correlation). It is important to note that resulted display will rely on inputs given on graph setting panel. If log layer(s) is applied, displayed graph will be automatically changed with more stress on slight difference between given sequences.

Third, input stream selection panel is used to manipulate the number of process on source code display. User can select which input stream he wants to relate with the number of process. In addition, user can also see the detail of input stream and check resulted output from executing targeted source code with given input. He is only required to press related buttons.

Fig. 6. Correlation graph display when theoretical sequence generates low correlation with the empirical one.
Fourth, source code display is used to display the relation between source code statements and their respective number of process. Such relation is displayed to provide further understanding toward the correlation between time complexity and empirical execution. The number of process for each statement is resulted from actual execution of given source code toward predefined input stream. Moreover, the total of all numbers of process is also provided at the bottom of source code display.

4 Evaluation

Five evaluation schemes are conducted: black-box evaluation, questionnaire-based evaluation, evaluation regarding minimum number of input stream, evaluation regarding non-asymptotic Big-O equation, and evaluation regarding the impact of involved statements. The first two schemes are related to J-CEL’s usability while the others are related to the characteristics of Pearson correlation for connecting theoretical sequence to empirical one.

4.1 Black-Box Evaluation

This evaluation scheme validates features provided by J-CEL. For each feature, its functionality is checked by simulating how user would use such feature. According to our findings, all J-CEL’s features work correctly as expected. They generate no error during the simulation.

4.2 Questionnaire-based Evaluation

This evaluation scheme validates whether J-CEL helps student for learning the correlation between Big-O equation and real program execution. Questionnaire surveys were distributed to 23 undergraduate students which had taken Algorithm Strategy course (i.e., a course where one of its topic is about algorithm time complexity) at our major. Each of them should rate 8 statements regarding to the use of J-CEL (see Table 1 for the detail of statements) in 4-points Likert scale (1 refers to strongly disagree, 2 refers to weakly disagree, 3 refers to weakly agree, and 4 refers to strongly agree). Further, rationale for the rating of each statement is also asked to be provided. To generate more objective result, these students had been asked to use J-CEL before they filled the survey.

Mean and standard deviation of questionnaire result can be seen on Fig. 7. Horizontal axis represents questionnaire statements while vertical axis represents resulted value. All statements are rated higher than 3 in average by students. Hence, it can be generally stated that students agree that J-CEL is helpful for learning the correlation between Big-O equation and real program execution. Among these statements, S3 generates the highest mean (i.e., 3.826) with the lowest standard deviation (i.e., 0.387). In other words, it can be stated that, without any dissents, students completely agree that correlation graph display helps student to determine which theoretical sequence is the most correlated one toward given empirical sequence. In contrast, S6 and S7 generate the lowest mean (i.e., 3.434). On the one hand, some students do not strongly agree with S6 since they feel that J-CEL should cover more learning aspects of algorithm time complexity (e.g., how to generate Big-O equation). On the other hand, some students do not strongly agree with S7 since they seldom learn through trial and error. It is important to note that S5 generates the highest standard deviation. In other words, numerous dissents are occurred toward S5.

When discovered further, some students feel that standard correlation graph display is
clear enough to see the correlation between theoretical and empirical sequence in detail.

**Table 1. Statements in Questionnaire Survey**

<table>
<thead>
<tr>
<th>ID</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Time complexity prediction on graph setting panel helps student to know time complexity of a particular algorithm.</td>
</tr>
<tr>
<td>S2</td>
<td>Correlation graph display helps student to correlate theoretical sequence with empirical one.</td>
</tr>
<tr>
<td>S3</td>
<td>Correlation graph display helps student to determine which theoretical sequence is the most correlated one toward given empirical sequence.</td>
</tr>
<tr>
<td>S4</td>
<td>The number of process resulted for each input stream helps student to understand which statements generate slow or fast processing time.</td>
</tr>
<tr>
<td>S5</td>
<td>The existence of logarithmic layer helps student to see the correlation between theoretical and empirical sequence in detail.</td>
</tr>
<tr>
<td>S6</td>
<td>J-CEL helps student to get a brief insight about the impact of time complexity.</td>
</tr>
<tr>
<td>S7</td>
<td>A mechanism to add time complexity equation helps student to understand the correlation between Big-O notation and real program execution through trial and error.</td>
</tr>
<tr>
<td>S8</td>
<td>Exception line helps student to only consider core operations for correlating Big-O notation and real program execution.</td>
</tr>
</tbody>
</table>

Fig. 7. Mean and standard deviation of questionnaire survey result.

All questionnaire statements are rated positively by our students. However, some statements are rated with 2 (weakly disagree) by at least one student. These statements are S1, S5, S6, S7, and S8. First, S1 is weakly disagreed by one respondent. He stated that he does not really understand how time complexity prediction works. Second, S5 is weakly disagreed by two respondents. One of them stated that it is unclear how many logarithmic layers required to provide clear view for some cases while the other stated that the difference between theoretical and empirical sequence is still large when only one logarithmic layer is applied. Third, S6 is weakly disagreed by one respondent. He stated that he could not relate the display with the correlation between
theoretical and empirical sequence. Fourth, S7 is weakly disagreed by two respondents. One of them stated that it is unclear about how to perform trial and error while another one stated that it is hard to write Big-O notation through given mechanism. Last, S8 is weakly disagreed by one respondent. He stated that it is impractical to put exception lines manually. It is important to note that, since the number of weakly-disagree rating is low, these rationales will be considered further on future work.

4.3 Evaluation regarding Minimum Number of Input Stream

This evaluation scheme enlists minimum number of input stream required for each well-known asymptotic algorithm time complexity. There are 7 time complexity equations involved: $O(1), O(log \ n), O(n), O(n \ log \ n), O(n^2), O(2^n)$, and $O(n!)$. Each equation will be featured with a source code that generates such complexity and ten input streams with various input sizes. The first five equations will use input size with $2^t$ growth and the last two equations will use input size with $t$ growth. $t$ refers to positive integer starting from 1 to 10. In other words, the first five equations will involve 2, 4, 8, 16, 32, 64, 128, 256, 512, and 1024 as their input sizes while the last two equations will involve 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 as their input sizes.

To evaluate minimum number of input stream, eleven evaluation schemes for each time complexity equation will be generated. One of them is generated by involving all input sizes, eight of them will be generated by iteratively removing the largest input size, and the other two will be generated by iteratively excluding in-between input sizes (i.e., input sizes that are placed at neither the beginning nor the end of the list). The latter mechanism is proposed to check whether the range between input sizes affects resulted correlation.

To provide clearer view about proposed mechanisms to generate evaluation schemes, suppose we have ten input streams with 2, 4, 8, 16, 32, 64, 128, 256, 512, and 1024 as their respective input size. Input sizes for each evaluation scheme can be seen in Table 2. The name for each scenario except default is prefixed with either M1 (for a mechanism that removes the largest input size) or M2 (for another mechanism). Such name is then followed by a positive integer representing the number of input stream. As seen in Table 2, it is clear that M1 excludes the largest input size each time its new scenario is generated. Further, M2 excludes input size in position 2, 5, 6, & 9 from default to generate M2-6 and excludes input size in position 2 & 5 from M2-6 to generate M2-4.

Minimum number of input stream for each time complexity equation is evaluated by executing its respective source code toward input streams provided by aforementioned scenarios and checking which scenario with the fewest input streams generates the highest correlation for correct time complexity equation. The result of that evaluation can be seen on Fig. 8. All equations are assigned correctly with the highest correlation on most scenarios. These equations except $O(1)$ are only assigned incorrectly on M1-2. We would argue that such finding is natural considering the number of input stream on such scenario is limited and these input streams are only slightly different to each other. Pearson correlation could not distinguish correct equation among the wrong ones on those conditions. It is important to note that $O(1)$ is still able to be detected on M1-2 since it is detected naively without the use of Pearson correlation.
Table 2. The Examples of Involved Input Sizes for Evaluation Schemes

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Involved Input Sizes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default</td>
<td>2, 4, 8, 16, 32, 64, 128, 256, 512, 1024</td>
</tr>
<tr>
<td>M1-9</td>
<td>2, 4, 8, 16, 32, 64, 128, 256, 512</td>
</tr>
<tr>
<td>M1-8</td>
<td>2, 4, 8, 16, 32, 64, 128, 256</td>
</tr>
<tr>
<td>M1-7</td>
<td>2, 4, 8, 16, 32, 64, 128</td>
</tr>
<tr>
<td>M1-6</td>
<td>2, 4, 8, 16, 32, 64</td>
</tr>
<tr>
<td>M1-5</td>
<td>2, 4, 8, 16, 32</td>
</tr>
<tr>
<td>M1-4</td>
<td>2, 4, 8, 16</td>
</tr>
<tr>
<td>M1-3</td>
<td>2, 4, 8</td>
</tr>
<tr>
<td>M1-2</td>
<td>2, 4</td>
</tr>
<tr>
<td>M2-6</td>
<td>2, 8, 16, 128, 256, 1024</td>
</tr>
<tr>
<td>M2-4</td>
<td>2, 16, 128, 1024</td>
</tr>
</tbody>
</table>

As seen in Fig. 8, scenarios generated by both M1 and M2 are favorable for each equation except when the number of input stream is 2. Hence, it can be stated that the range between input size does not affect correlation result and generated correlation result will be correct as long as its number of input stream is higher than 2 with reasonable range between input streams (in our case, we assign $2^t$ growth for low time complexity equations and $t$ growth for high time complexity equations where $t$ refers to positive integer).

![Stacked Line for Time Complexity Equations](image)

Fig. 8. Stacked line for time complexity equations; each equation will be assigned with 1 if the highest correlation is assigned correctly to that equation on given scenario. Otherwise, it will be assigned with 0.
4.4 Evaluation regarding Non-Asymptotic Big-O Equation

This evaluation scheme validates whether resulted Pearson correlation is increased when non-asymptotic Big-O equation is incorporated and the difference between non-asymptotic and asymptotic equation is considerably large. To do so, time complexity equations defined for evaluating the minimum number of input stream except $O(1)$ will be used. $O(1)$ is excluded since determining such complexity does not require Pearson correlation. For each equation, two scenarios are proposed: asymptotic and non-asymptotic scenario. The former one refers to a condition where inputted equation is the non-asymptotic one while the latter one refers to a condition where inputted equation is the asymptotic one. It is important to note that non-asymptotic scenario is implemented by utilizing J-CEL’s feature to add new time complexity equation.

Pearson correlation result for both scenario toward all involved time complexity equations can be seen in Fig. 9. Non-asymptotic scenario generates higher or similar correlation value when compared to the asymptotic one. We would argue that similar correlation value between both scenarios on several equations is natural considering the difference between equation used in both scenarios is extremely small. Consequently, it still can be stated that resulted Pearson correlation is increased when non-asymptotic Big-O equation is incorporated and the difference between non-asymptotic and asymptotic equation is considerably large.

![Pearson Correlation between Asymptotic and Non-Asymptotic Scenario](image)

Fig. 9. Pearson correlation between asymptotic and non-asymptotic scenario for each equation. The highest possible correlation for each scenario is 1.

4.5 Evaluation regarding The Impact of Involved Statements

This evaluation scheme validates whether resulted Pearson correlation is increased when only statements related to core operations are involved. Time complexity equations defined for evaluating the minimum number of input stream except $O(1)$ will be used. For each equation, two scenarios are proposed: standard and only-core-operations scenario. The former one refers to a condition where all statements are
involved while the latter one refers to a condition where only statements related to core operations are involved. Only-core-operations scenario is implemented by utilizing J-CEL’s exception lines.

Pearson correlation result for both scenarios toward all involved time complexity equations can be seen in Fig. 10. Only-core-operations scenario generates higher correlation value when compared to the standard one. Such finding is natural since only-core-operations scenario excludes input and output statements from each case, resulting more accurate empirical sequence. Hence, it can be stated that resulted Pearson correlation is increased when only statements related to main operations are involved.

Fig. 10. Pearson correlation between standard and only-core-operations scenario for each equation. The highest possible correlation for each scenario is 1.

5 Conclusion and Future Work

In this paper, an educational tool for learning algorithm time complexity by examples (i.e., J-CEL) is proposed. We would argue that such tool is more practical to be used when compared to Complexitor [3]; our tool can be used without deep technical knowledge about programming language. According to our evaluation, students feel that J-CEL is helpful for learning the correlation between Big-O equation and real program execution. Further, the use of Pearson correlation in J-CEL shows a promising result. It always generates the highest correlation for correct time complexity equation as long as it involves more than two input streams with acceptable input range size (in our case, we assign $2^t$ growth for low time complexity equations and $t$ growth for high time complexity equations; where $t$ refers to positive integer). Its resulted correlation even goes higher when either non-asymptotic equation is incorporated or statements unrelated to core operations are excluded. For future work, we plan to evaluate the impact of J-CEL through quasi experiment [28] on Algorithm Strategy course (i.e., a course where one of its topic is about algorithm time complexity). Moreover, we also plan to cover negative feedbacks resulted from questionnaire survey.
References

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